MOMENTUM PRINCIPLE

The Momentum Equation for Cartesian Coordinates

X-direction:

$$\sum F_X = \frac{d}{dt}\int_{cv} v_X \rho dQ + \sum_{CS}(\dot{m}v)_{outX} - \sum_{CS}(\dot{m}v)_{inX}$$

Y-direction:

$$\sum F_Y = \frac{d}{dt}\int_{cv} v_Y \rho dQ + \sum_{CS}(\dot{m}v)_{outY} - \sum_{CS}(\dot{m}v)_{inY}$$

Z-direction:

$$\sum F_Z = \frac{d}{dt}\int_{cv} v_Z \rho dQ + \sum_{CS}(\dot{m}v)_{outZ} - \sum_{CS}(\dot{m}v)_{inZ}$$

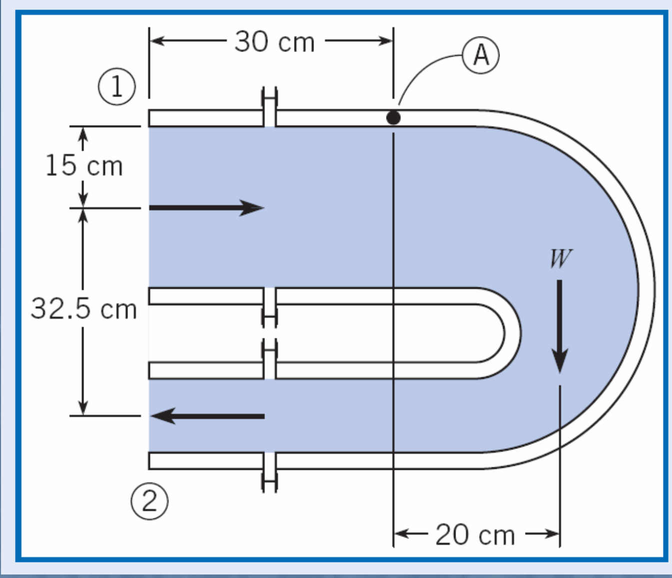
Moment - of - Momentum Equation

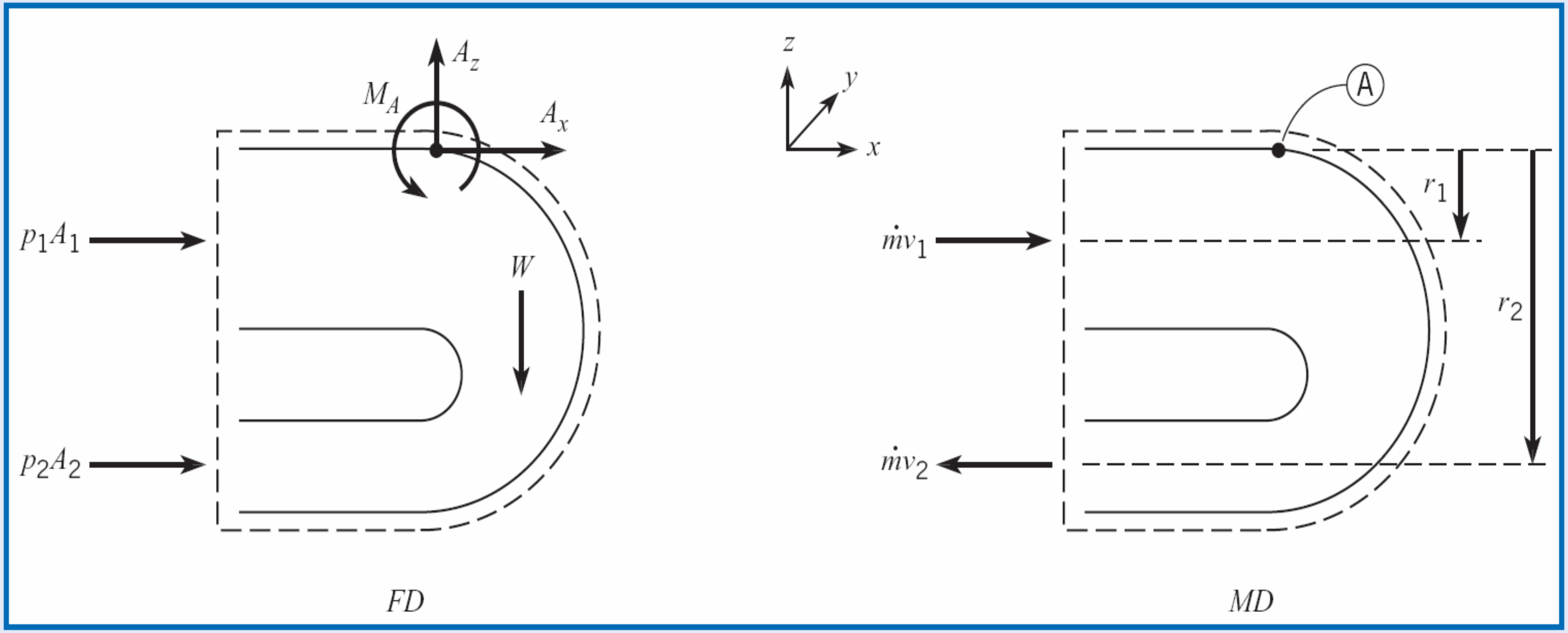
$$\sum M = \frac{d}{dt}\int_{CV}(r \times v)\rho dQ + \sum_{CS} r \times (\dot{m}v)_{out} - \sum_{CS} r \times (\dot{m}v)_{in}$$

Example (6.12)

Find: The moment the support system must resist?

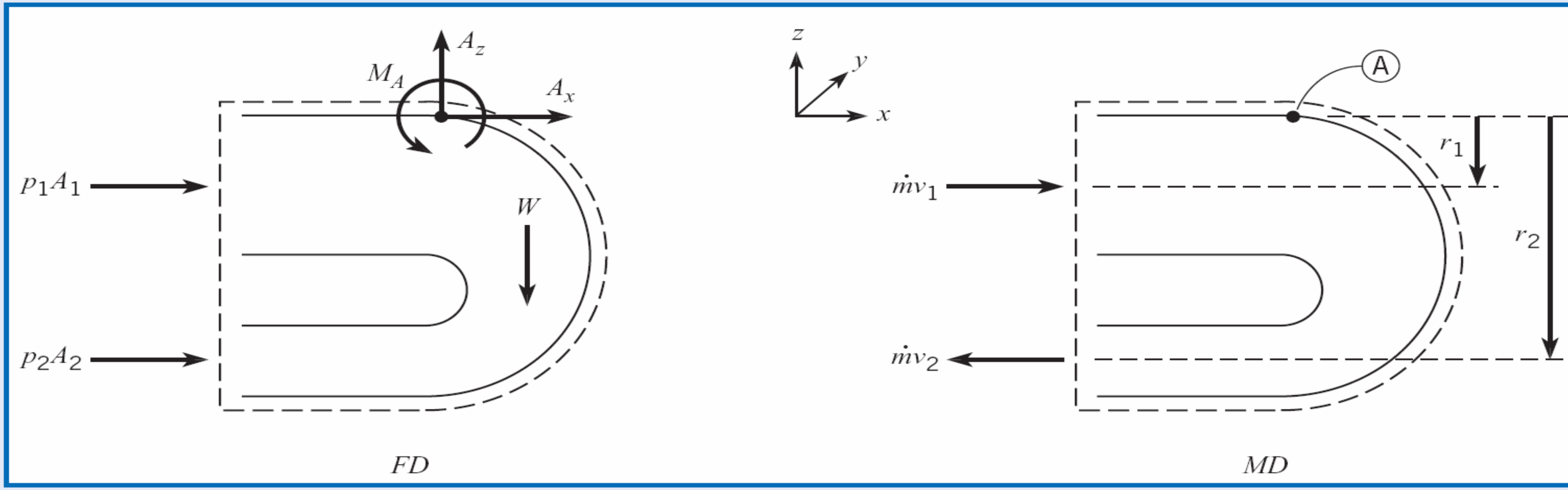
The bend is supported at Point (A)





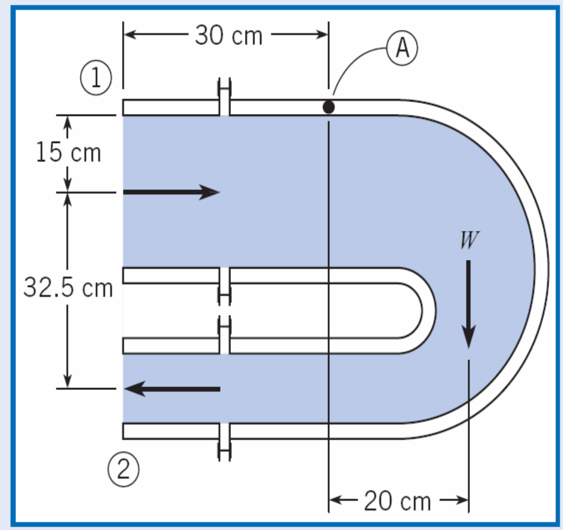
ANALYSIS:

The momentum accumulation = $\dfrac{d}{dt}\int_{cv} v_Z \rho dQ = 0$



From the force diagram,

$$\sum M = -(M_A + 0.15 p_1 A_1 + 0.475 p_2 A_2 - 0.2W)$$



From the momentum diagram,

$$\sum M = \sum_{CS} r_{out} \times (\dot{m}v)_{out} - \sum_{CS} r_{in} \times (\dot{m}v)_{in}) = r_2 \times (\dot{m}v_2) + r_1 \times (\dot{m}v_1)$$

Equating the above Eqns. we have

$$\sum M = -(M_A + 0.15 p_1 A_1 + 0.475 p_2 A_2 - 0.2W) = \dot{m}(r_2 v_2 + r_1 v_1)$$

$$0.15 p_1 A_1 = (0.15\ \text{m})(150 \times 1000\ \text{N/m}^2)(\pi \times 0.3^2/4\ \text{m}^2)$$
$$= 1590\ \text{N} \cdot \text{m}$$

$$0.475 p_2 A_2 = (0.475\ \text{m})(59.3 \times 1000\ \text{N/m}^2)(\pi \times 0.15^2/4\ \text{m}^2)$$
$$= 497.8\ \text{N} \cdot \text{m}$$

The mass flow rate is

$$\dot{m} = \rho Q = (1000\ \text{kg/m}^3)(0.25\ \text{m}^3/\text{s}) = 250\ \text{kg/s}$$

The net angular-momentum outflow is

$$\dot{m}(r_2 v_2 + r_1 v_1) = (250\ \text{kg/s})(0.475 \times 14.15 + 0.15 \times 3.54)(\text{m}^2/\text{s})$$
$$= 1813\ \text{N} \cdot \text{m}$$

The moment exerted by the support is

$$M_A = -0.15p_1A_1 - 0.475p_2A_2 + 0.2W - \dot{m}(r_2v_2 + r_1v_1)$$
$$= -(1590 \text{ N} \cdot \text{m}) - (497.8 \text{ N} \cdot \text{m})$$
$$+ (0.2 \text{ m} \times 1481 \text{ N}) - (1813 \text{ N} \cdot \text{m})$$
$$= -3.60 \text{ kN} \cdot \text{m} \qquad \triangleleft$$

Thus, a moment of 3.60 kN · m acting in the (j) or clockwise direction is needed to hold the bend stationary. Stated differently, the support system must be designed to withstand a counterclockwise moment of 3.60 kN · m.

Example (6.13)

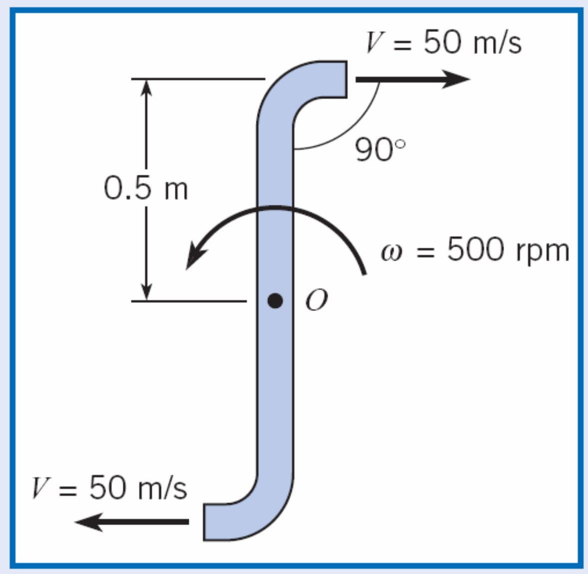
Determine the power produced by the turbine?

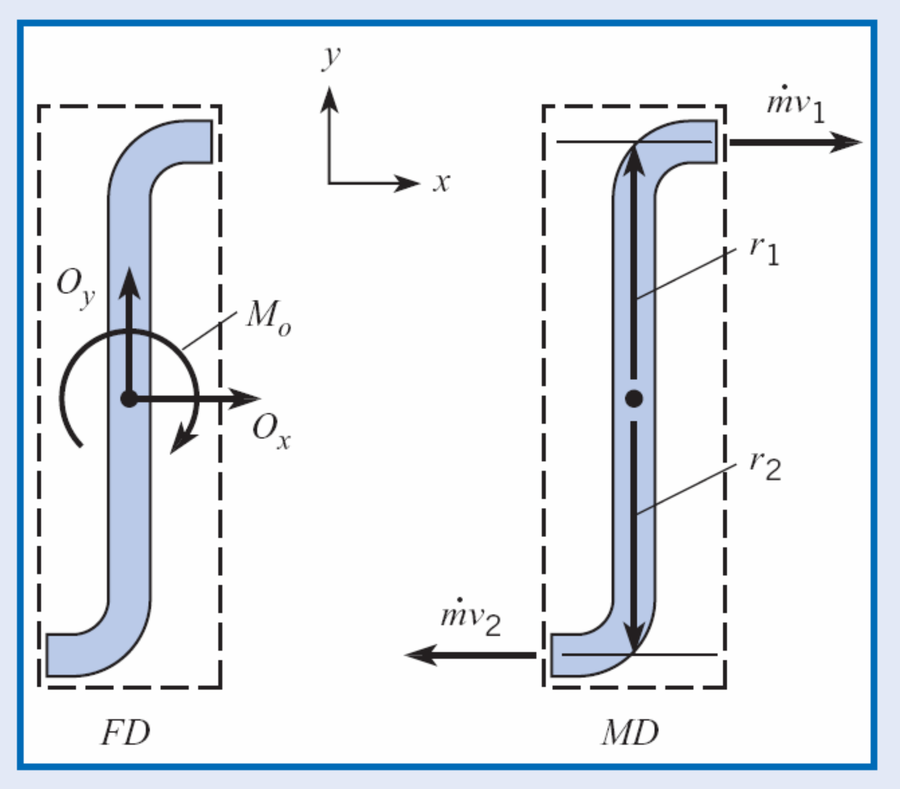
$\omega = 500\, rpm$

$A_1 = A_2 = 10\, cm^2$

$v_1 = v_2 = 50\, m/\sec$

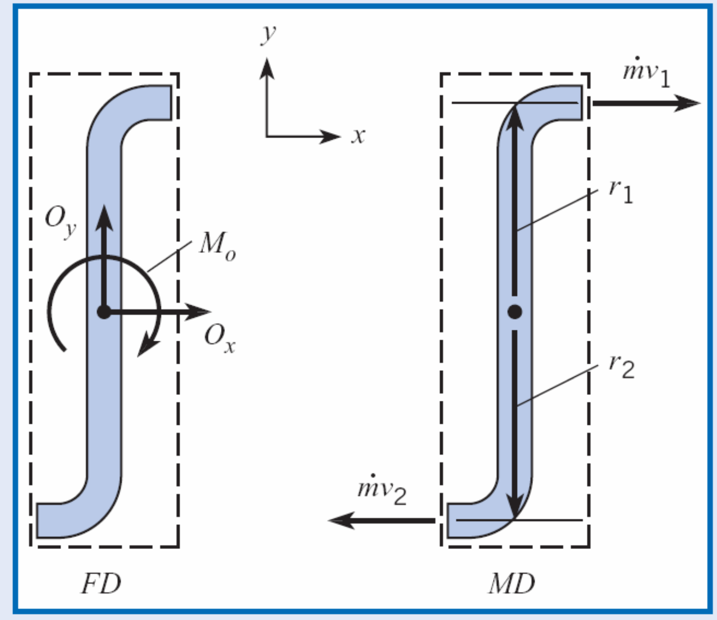
$\rho_{water} = 1000\, kg/m^3$





ANALYSIS:

1. We select a moving control volume.
2. We select the point (O) as the moment center.



From the force diagram, $\sum M_o = -M_o k$

From the momentum diagram,

Zero

$$\sum M = \sum_{CS} r_{out} \times (\dot{m}v)_{out} - \sum_{CS} r_{in} \times (\dot{m}v)_{in}) = r_2 \times (\dot{m}v_2) + r_1 \times (\dot{m}v_1)$$

Equating the above Eqns. , we have

$$M_0 = r_2 \times (\dot{m}v_2) + r_1 \times (\dot{m}v_1) = 2(\dot{m}rv)$$

$$(r_1 = r_2, \quad v_1 = v_2)$$

The mass flow rate term uses the speed V that is relative to the control surface,

$$\dot{m} = \rho A V$$

Combining terms gives

$$M_o = 2r\rho AV(V - \omega r)$$

Mass flow rate is

$$\dot{m} = \rho AV = (1000\ \text{kg/m}^3)(0.001\ \text{m}^2)(50\ \text{m/s}) = 50\ \text{kg/s}$$

The rotational speed is $\omega = 500\ \text{rpm} = 52.4\ \text{rad/s}$, and so the moment is

$$\begin{aligned} M_o &= 2r\dot{m}[V - \omega r] \\ &= 2(0.5\ \text{m})(50\ \text{kg/s})[50\ \text{m/s} - (52.4\ \text{rad/s})(0.5\ \text{m})] \\ &= 1190\ \text{N}\cdot\text{m} \end{aligned}$$

The power produced by the turbine is the product of the moment and the angular speed:

$$\begin{aligned} P = M_o\omega &= (1190\ \text{N}\cdot\text{m})(52.4\ \text{rad/s}) \\ &= 62.4\ \text{kW} \end{aligned}$$

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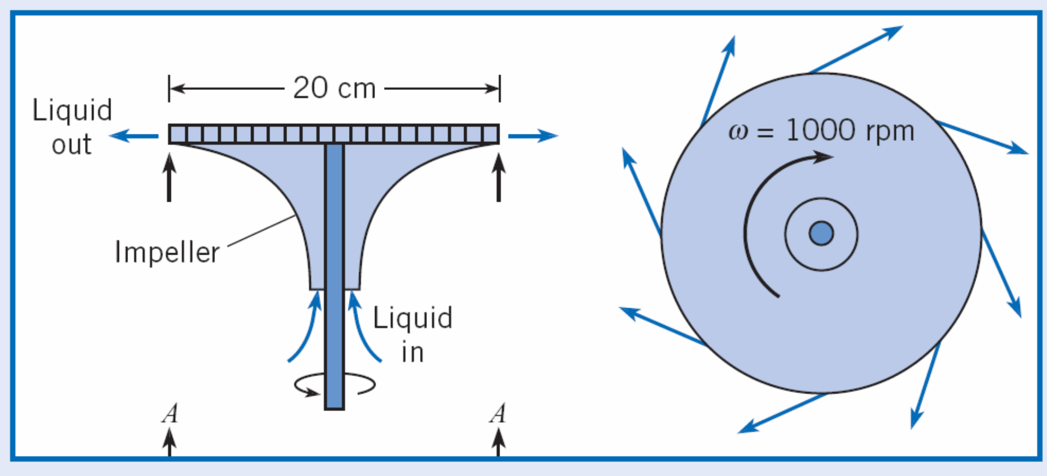
Example (6.14)

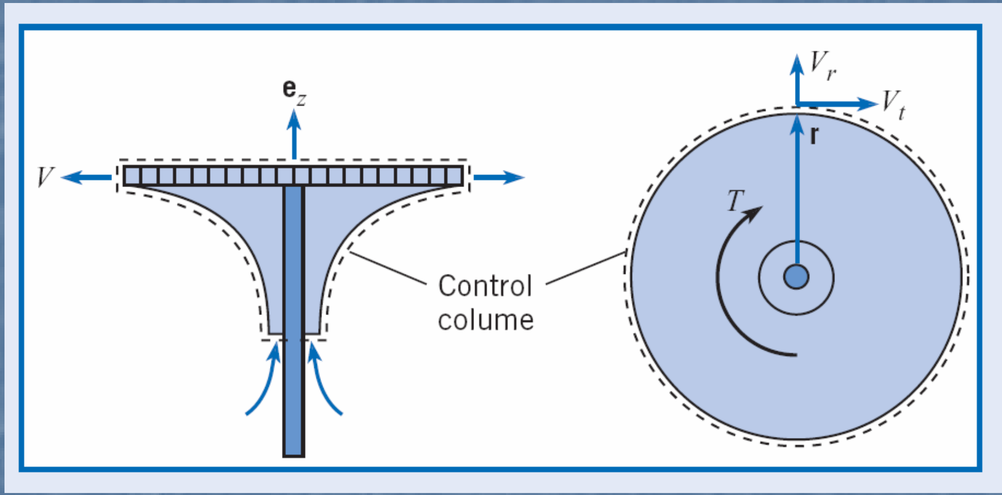
Find:

The torque and power required to operate the pump?

Given:

$\omega = 1000\, rpm, \dot{Q} = 400\, Lpm, d_{impeller} = 20\, cm$





ANALYSIS:

- The torque (T) is applied in the positive z-direction.
- The tangential component of velocity at the inlet = zero.

The moment of momentum accumulation $\frac{d}{dt}\int_{cv}(r\times v)\rho dQ = 0$ (Flow is steady)

Applying the moment of momentum equation,

$$\sum M = Te_z = \sum_{CS} r_{out}\times(\dot{m}v)_{out} - \sum_{Ci} r_{in}\times(\dot{m}v)_{in}) = r_o\times(\dot{m}v_{ot}) - r_i\times(\dot{m}v_{it})$$

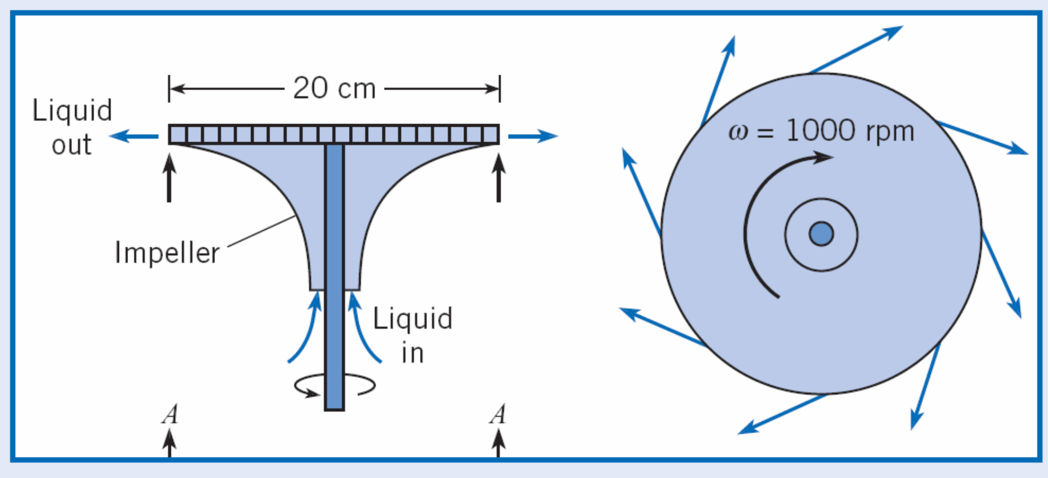
$$\sum M = Te_z = r_o\times(\dot{m}v_{ot}) - 0 \qquad (v_{it} = 0)$$

$$\sum M = Te_z = \dot{m}\omega r_0^{\,2} \qquad As\ v_{ot} = \omega r_o, \quad r_o = \frac{D_o}{2} \qquad \dot{m} = \rho Q$$

Torque produced $$Te_z = (\rho Q)\omega\frac{D_o^{\,2}}{4} \qquad \omega = \frac{2\pi N}{60} \qquad N = rpm$$

Power produced, $P = T\,\omega$

Example (6.14)



$$Te_z = (\rho Q)\omega \frac{D_o^{\ 2}}{4}$$

where D is the outlet diameter of the impeller. The mass flow rate is

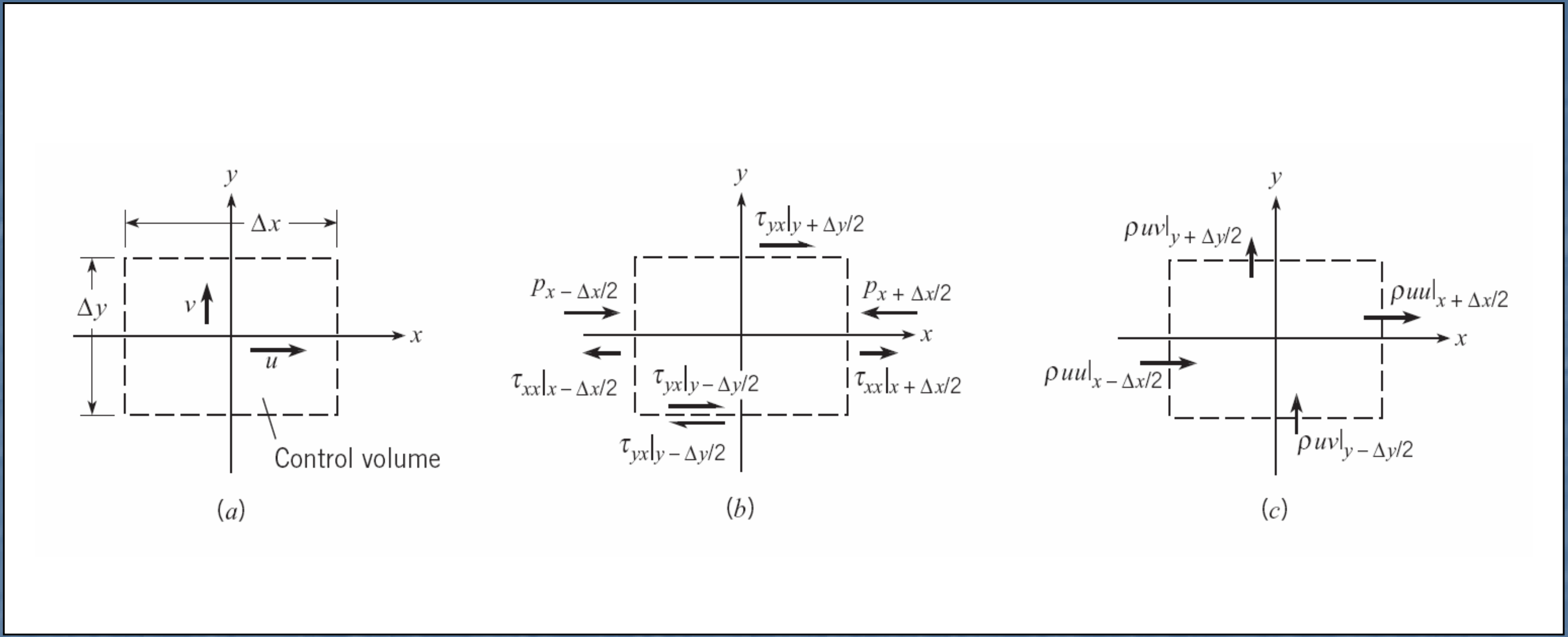
$$\rho Q = 1000 \text{ kg/m}^3 \times 400 \ l/\text{min} \times (1/60) \text{ min/s} \times 10^{-3} \text{m}^3/l$$
$$= 6.67 \text{ kg/s}$$

and the rotational rate is

$$\omega = 1000 \text{ rev/min} \times 2\pi \text{ rad/rev} \times (1/60) \text{ min/s}$$
$$= 104.7 \text{ rad/s}$$

Power produced, $P = T\,\omega$

Navier – Stokes Equation



The differential equation for momentum at a point in the x-direction

$$\rho \frac{Du}{Dt} = -\frac{\partial p}{\partial x} + \mu\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right] + \rho g_x$$

The differential equation for momentum at a point in the y-direction

$$\rho \frac{Dv}{Dt} = -\frac{\partial p}{\partial y} + \mu\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right] + \rho g_y$$

END OF LECTURE (7)